Beamforming Arrays with Faulty Sensors in Dynamic Environments

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Abstract This paper addresses the problem of beamforming a uniform linear array when some of the receive elements are not operational. Bad sensors are often handled by either zeroing or interpolating the faulty elements prior to conventional beamforming. While zeroing faulty elements prior to conventional beamforming is the simplest approach, it often results in undesirably high beamformer sidelobes. Alternatively, minimum mean-square error (MMSE) interpolation of the missing data is not explicitly aimed at minimizing post-interpolation leakage of strong interference components into otherwise quiet directions. While true minimum variance (MV) adaptive beamforming is the optimal solution given long observation times, for large arrays in highly dynamic environments, severely limited snapshot support poses a difficult trade-off between desired interference suppression versus unwanted signal cancellation.

In this work, we propose an alternative approach for conventional beamforming with faulty sensors based on adaptively synthesizing complete array data snapshots which minimize post-interpolation leakage into quiet directions subject to a constraint that the solution is sufficiently close to the data measured at the working elements. After reconstruction of the complete array data snapshots, computationally efficient conventional beamforming can be performed to both estimate the noise field directionality as well as produce time-series output for further temporal analysis.

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# Beamforming Sensor Arrays with Faulty Sensors

OBJECTIVE: To mitigate sidelobe degradation due to sensor outages in large sonar arrays.

BACKGROUND:

- Target detection in interference-dominated littoral regions depends critically on achieving low beamformer sidelobe levels. Sensor failures can seriously degrade the array pattern of conventional beamformers.

- While zeroing faulty elements prior to conventional beamforming is a common remedy, it often results in undesirably high beamformer sidelobes.

- Adaptive beamforming using only the good sensors provides asymptotically optimal array gain, however highly dynamic littoral environments make achieving this asymptotic performance a challenging problem.

- We investigate adaptive channel interpolation and compensation techniques which mitigate faulty sensors on a snapshot-by-snapshot basis and are thus applicable to large arrays in dynamic environments.

- Minimum mean-square error (MMSE) interpolation of bad elements is an obvious approach but doesn't explicitly minimize leakage of strong interference components into otherwise quiet directions.

- Spatial autoregressive (AR) methods have been previously proposed for array interpolation by Swingler and Walker (1988) but are not designed to handle spatially inhomogeneous multipath interferers.

- In this work, we consider a linear inverse theoretic framework for mitigating the effects of faulty sensors.

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Motivation for Mitigating Faulty Array Elements

- Bearing-time record (BTR) from real 30-element passive sonar towed-array data. Full-array BTR (left). Degraded with 6 zeroed channels (right).

- Note degradation in sidelobe response complicates weak target detection.

Bearing Time Record - Full Array

Bearing Time Record with faulty elements
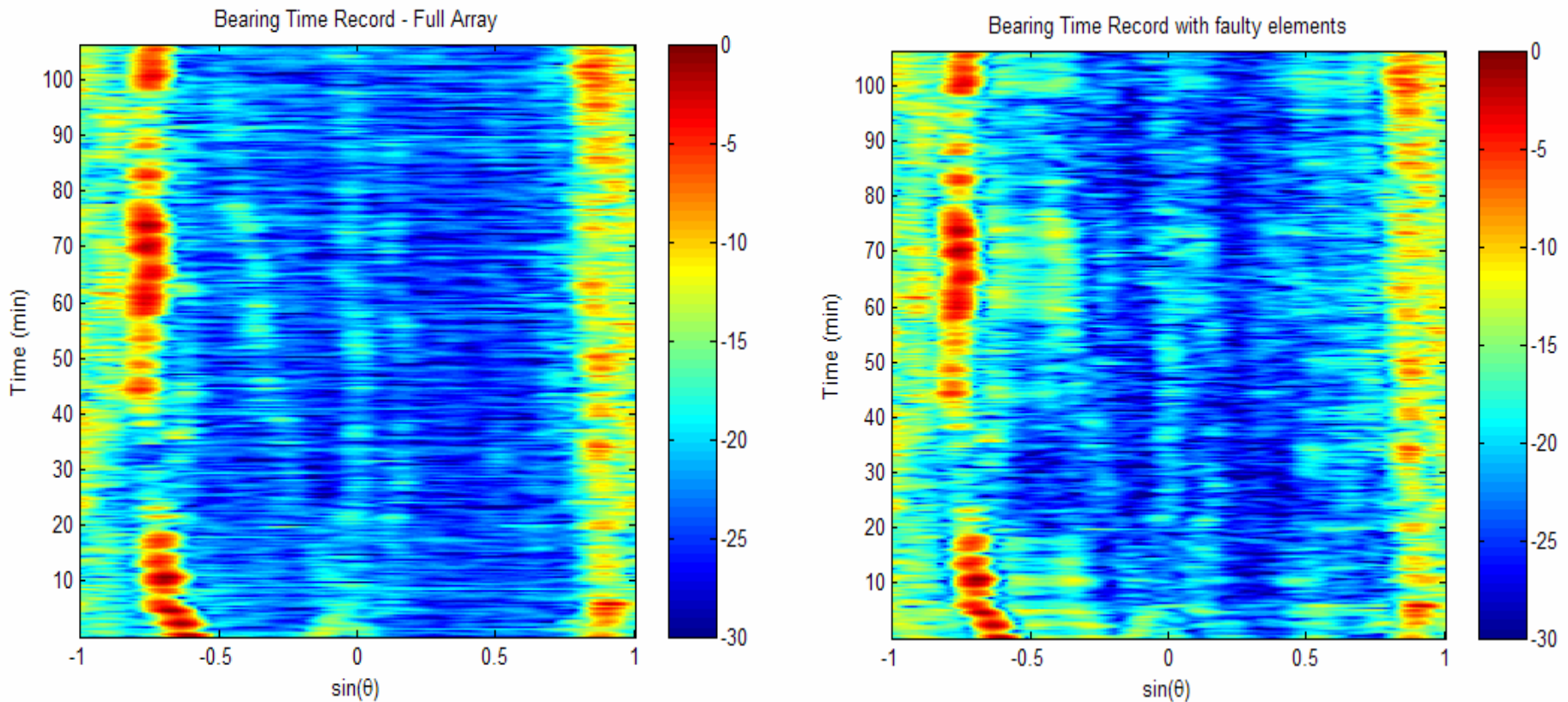
Array Data Modeling with Missing Sensors

- Model the output from an *N*-element uniform linear array as zero-mean complex snapshots $\mathbf{x}_k$ with covariance matrix consisting of signal, interference, and noise:

$$\mathbf{R}_x = \sigma_s^2 d(\theta_s)d(\theta_s)^+ + \mathbf{APA}^+ + \sigma_n^2\mathbf{I}$$

where $d(\theta_s)$ and σ_s^2 are the signal wavefront and power, $\mathbf{APA}^+$ defines the unknown interference covariance, and $\sigma_n^2\mathbf{I}$ represents the white noise component.

- Zeroing missing sensors corresponds to using a covariance matrix with correspondingly zeroed columns/rows e.g.

$$\begin{pmatrix} r_{1,1} & r_{1,2} & 0 & r_{1,4} & r_{1,5} \\ r_{2,1} & r_{2,2} & 0 & r_{2,4} & r_{2,5} \\ 0 & 0 & 0 & 0 & 0 \\ r_{4,1} & r_{4,2} & 0 & r_{4,4} & r_{4,5} \\ r_{5,1} & r_{5,2} & 0 & r_{5,4} & r_{5,5} \end{pmatrix}$$

- Classical interpolation methods can be thought of as "filling in" missing covariance matrix elements using Toeplitz approximation and then designing a linear filter for MMSE interpolation on a snapshot-by-snapshot basis.

Covariance Matrix Estimation with One Snapshot

- Covariance matrix estimation with as little as a single snapshot of data typically requires imposing a Toeplitz structure (i.e. spatially stationary field).

- A straightforward Toeplitz covariance matrix estimate can be computed by using the weighted projected covariance estimator proposed by Barton and Smith (1997) which uses biased average diagonals of the sample covariance matrix.

- For a fully populated array, this method is equivalent to computing a Toeplitz correlation matrix with the biased autocorrelation sequence of the array data.

$$r(k) = \frac{1}{N} \sum_{n=k}^{N-1} x(n) x^*(n-k) \qquad \hat{\mathbf{R}} = toeplitz(\mathbf{r})$$

- Toeplitz approximation models the interference subspace as a linear combination of uncorrelated plane wave returns corresponding to interference directions:

$$\mathbf{A}\mathbf{P}\mathbf{A}^+ + \sigma_n^2\mathbf{I} \cong \hat{\mathbf{A}}\hat{\mathbf{P}}\hat{\mathbf{A}}^+ + \mathbf{\Phi}_{\mathbf{min}}\mathbf{\Lambda}_{\mathbf{min}}\mathbf{\Phi}_{\mathbf{min}}^+ \text{ where } \hat{\mathbf{A}}^+\mathbf{\Phi}_{\min} = \mathbf{0}$$

and $\hat{\mathbf{P}}$ and $\Lambda_{\min}$ are positive diagonal matrices.

- More sophisticated methods for structured covariance matrix estimation have been developed by numerous researchers (e.g. Jansson and Ottersten, 2000, Robey and Fuhrmann, 1991, Stoica and Li, 1999, Cadzow, 1988, Williams and Johnson, 1988,) but typically involve greater computational complexity.

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Minimum MSE Interpolation of Faulty Sensors

- A natural solution for handling faulty channels is to interpolate them using a linear combination of the good sensor outputs.

- The linear filter, $\mathbf{T}_b$, which minimizes mean-square error (MSE), i.e.

$$\min_{\mathbf{T_b}} \ \left\| \mathbf{x_b} - \mathbf{T_b x_g} \right\|^2 \text{ is given by } \ \mathbf{T_b} = \mathbf{R_{bg} R_{gg}^{-1}}$$

  where $\mathbf{x_g} = \mathbf{L}^+ \mathbf{x}$ are the *N-m* good channels and $\mathbf{x_b} = \mathbf{K}^+ \mathbf{x}$ are the *m* bad channels.

- A single snapshot estimate of the cross-correlation between faulty and good channels is obtained from the Toeplitz covariance matrix estimate of the full array data.

- The Minimum MSE (MMSE) interpolated full array data snapshot can be written as a linear transformation matrix $\mathbf{T_m}$ operating on the original data snapshots:

$$\mathbf{q_m} = \mathbf{T_m x} \quad \text{where } \mathbf{T_m} = (\mathbf{L} + \mathbf{K T_b}) \mathbf{L^H}$$

- Minimizing channel-wise MSE does not explicitly minimize sidelobe leakage of interferers into quiet directions and thus only indirectly mitigates impact of bad channels.

- Alternative ALPINEX method of uses AR modeling of interferers and averaged forward and backward linear prediction of faulty channels (Swingler and Walker, 1998).

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Adaptive Channel Compensation (ACC)

Idea: Adaptively reconstruct full-array snapshot, **q,** such that interference leakage into nominally quiet directions is minimized.

- The proposed adaptive channel compensation (ACC) method minimizes interference leakage into quiet plane-wave directions subject to the constraint that the data distortion at the good elements is within some tolerance, ε i.e.

$$\min_{\mathbf{q}} \ (\mathbf{q}^{+}\boldsymbol{\Phi}\boldsymbol{\Phi}^{+}\mathbf{q}) \ \text{subject to the constraint} \ \left\| \mathbf{L}^{\mathbf{H}}\mathbf{q} - \mathbf{x_g} \right\|_{2}^{2} < \varepsilon$$

$$\text{Solution:} \quad \mathbf{q} = (\boldsymbol{\Phi}\boldsymbol{\Phi}^{+} + \mu\,\mathbf{L}\mathbf{L}^{+})^{-1}\mu\,\mathbf{L}\mathbf{x_g}$$

where the columns of $\boldsymbol{\Phi}$ are the sub-dominant eigenvectors

of the Toeplitz-approximated $\hat{\mathbf{R}}_{\mathbf{x}}$.

- Choice of ε (which is inversely related to μ) determines the trade-off between signal wavefront distortion and mitigation of interference leakage.

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ACC as the Solution to a Linear Inverse Problem

- Adaptive channel compensation can be interpreted as inversion for the complete data using the linear operator **T** operating on the corrupted data snapshot:

$$\mathbf{T} = (\boldsymbol{\Phi}\boldsymbol{\Phi}^+ + \mu\,\mathbf{LL}^+)^{-1}\mu\,\mathbf{LL}^+$$

- Decomposing **T** in terms of the dominant $\boldsymbol{\Phi}_{\mathbf{max}}\boldsymbol{\Phi}_{\mathbf{max}}^+$ and sub-dominant $\boldsymbol{\Phi}\boldsymbol{\Phi}^+$ subspaces of $\hat{\mathbf{R}}_{\mathbf{x}}$ gives:

$$\mathbf{T} = (\boldsymbol{\Phi}_{\mathbf{max}}\boldsymbol{\Phi}_{\mathbf{max}}^+ + (\frac{\mu}{1+\mu})\boldsymbol{\Phi}\boldsymbol{\Phi}^+)(\mathbf{I} - \mathbf{K}(\mathbf{K}^+\boldsymbol{\Phi}\boldsymbol{\Phi}^+\mathbf{K})^{-1}\mathbf{K}^+\boldsymbol{\Phi}\boldsymbol{\Phi}^+)\,\mathbf{LL}^+$$

  where $\mathbf{K}^+$ and $\mathbf{L}^+$ select the faulty and good elements and $\mathbf{LL}^{\mathbf{H}} + \mathbf{KK}^{\mathbf{H}} = \mathbf{I}$

- Effect of ACC is to make interferers look more like plane-waves across the full-array which can then be effectively nulled by conventional beamformer shading.

- Signals in the subdominant subspace are not interpolated but suppressed by a factor of $\mu/(1+\mu)$

- Choice of $\mu = 10$ empirically found to provide good compensation of faulty channels with negligible signal distortion.

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Comparing ACC vs. Adaptive Beamforming

- Owsley (1984) previously studied asymptotic array gain (AG) improvement achieved by adaptive beamforming versus conventional processing in terms of $R_g = AG_{Optimum} / AG_{CB}$

- Using this metric to compare conventional beamforming with ACC vs. adaptive beamforming (ABF) gives:

$$R_g = \frac{\text{SINR}_o}{\text{SINR}_a} = 1 + \frac{(\text{INR}_{out})^2}{1 + \text{INR}_{out}} \text{L}_i (1 - \text{L}_i)$$

where $\text{L}_i = \left| \dfrac{\mathbf{d}(\theta)^{\mathbf{H}} \mathbf{T} \tilde{\mathbf{d}}(\phi)}{N} \right|^2$

Array gain improvement ratio for optimum MVDR beamformer

- For sidelobe interference, $L_i \square 1$, and thus ACC only slightly under-performs asymptotically optimum beamformer for moderate INR's.

- ACC expected to have the advantage in non-asymptotic scenarios where ABF nulling degraded by poor eigenvector estimation.
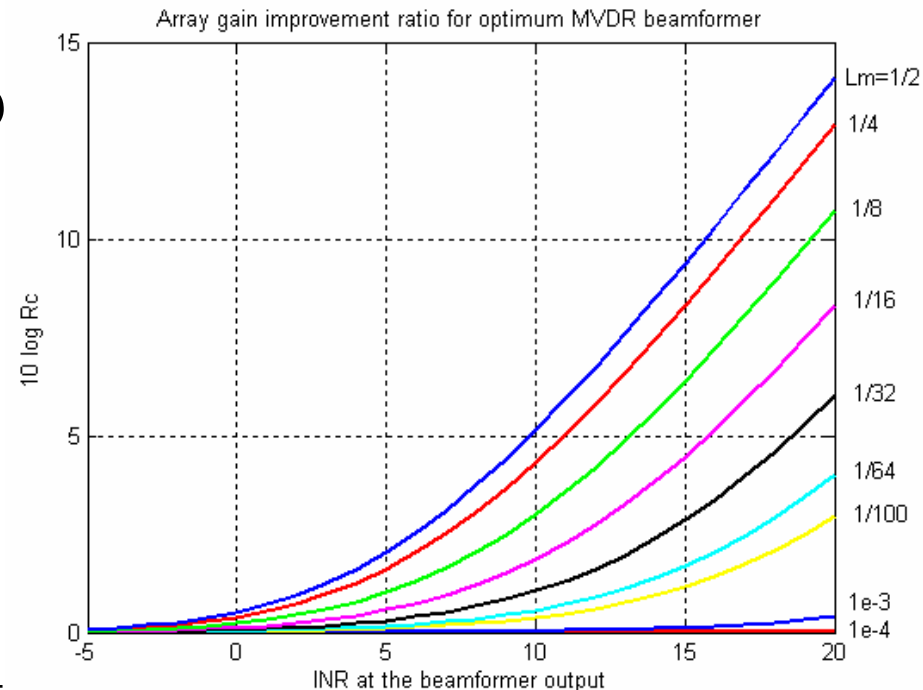
Example Beamformer Array Pattern Degradation

- Simulation example of the array patterns (right) from a Hamming-shaded 30-element array data with and without 6 missing elements (below).

- Note severe sidelobe degradation due to zeroed elements.
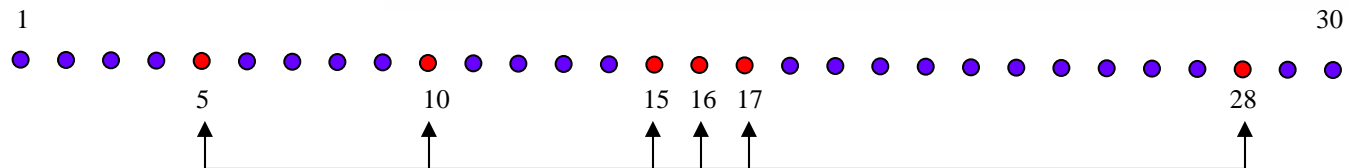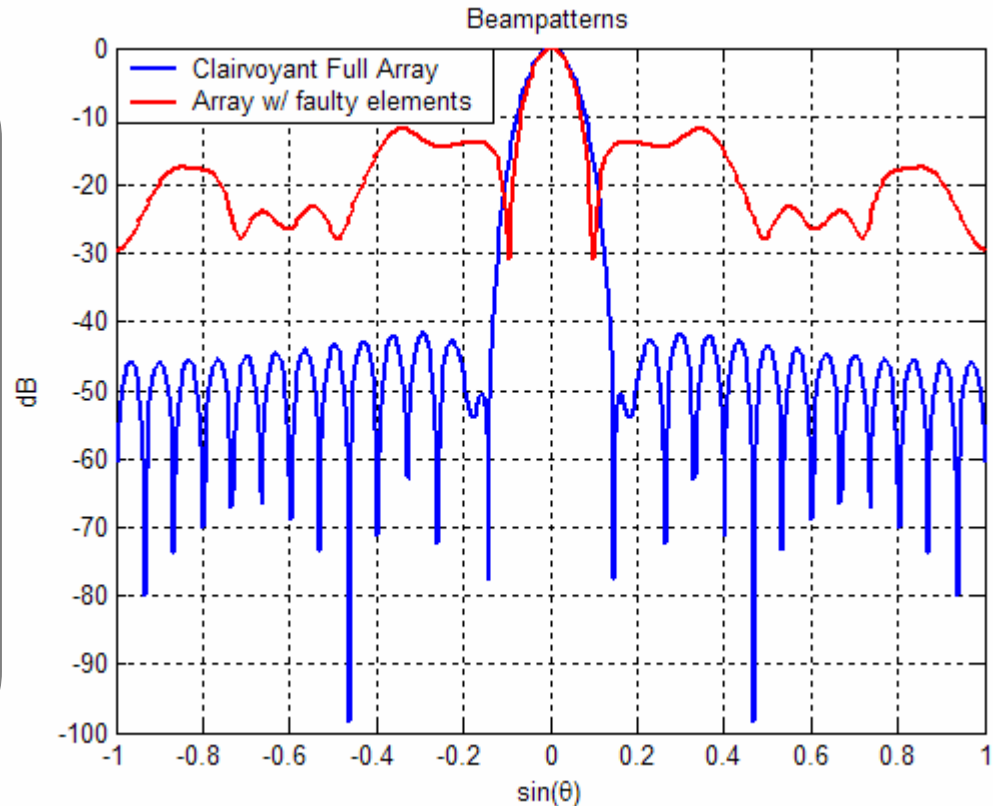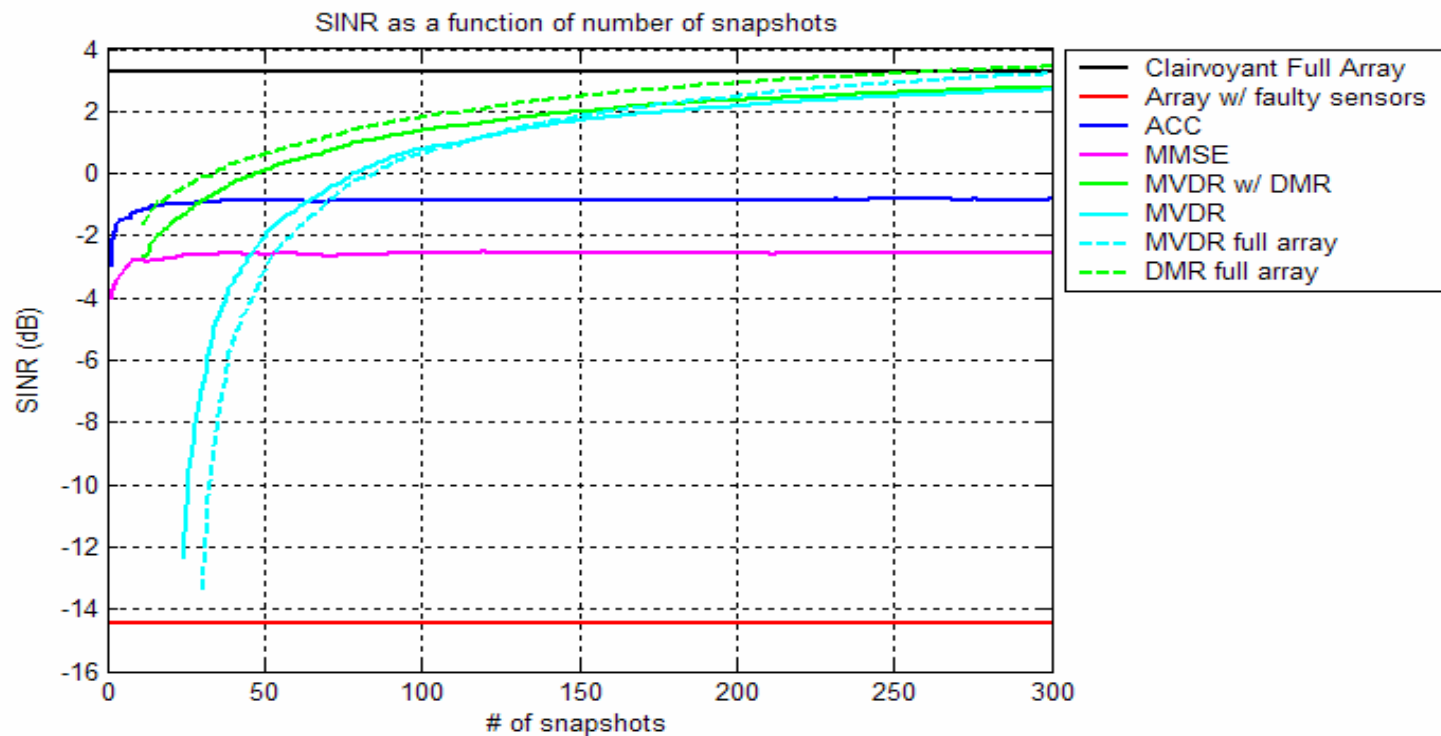

Faulty Elements

Simulated Single Snapshot Field Directionality Map

- Comparison of clairvoyant full array, zeroed conventional, ALPINEX, and MMSE methods versus adaptive channel compensation (ACC) for a 30 element array with a 0 dB target, 20 dB correlated interference at $\sin(\theta) = 0$ and -0.2 and 10 dB uncorrelated plane-wave interference at $\sin(\theta) = .47$

- ACC with Hamming beamforming clearly unmasks the target and achieves lowest sidelobe levels for this scenario of 6 faulty sensors.
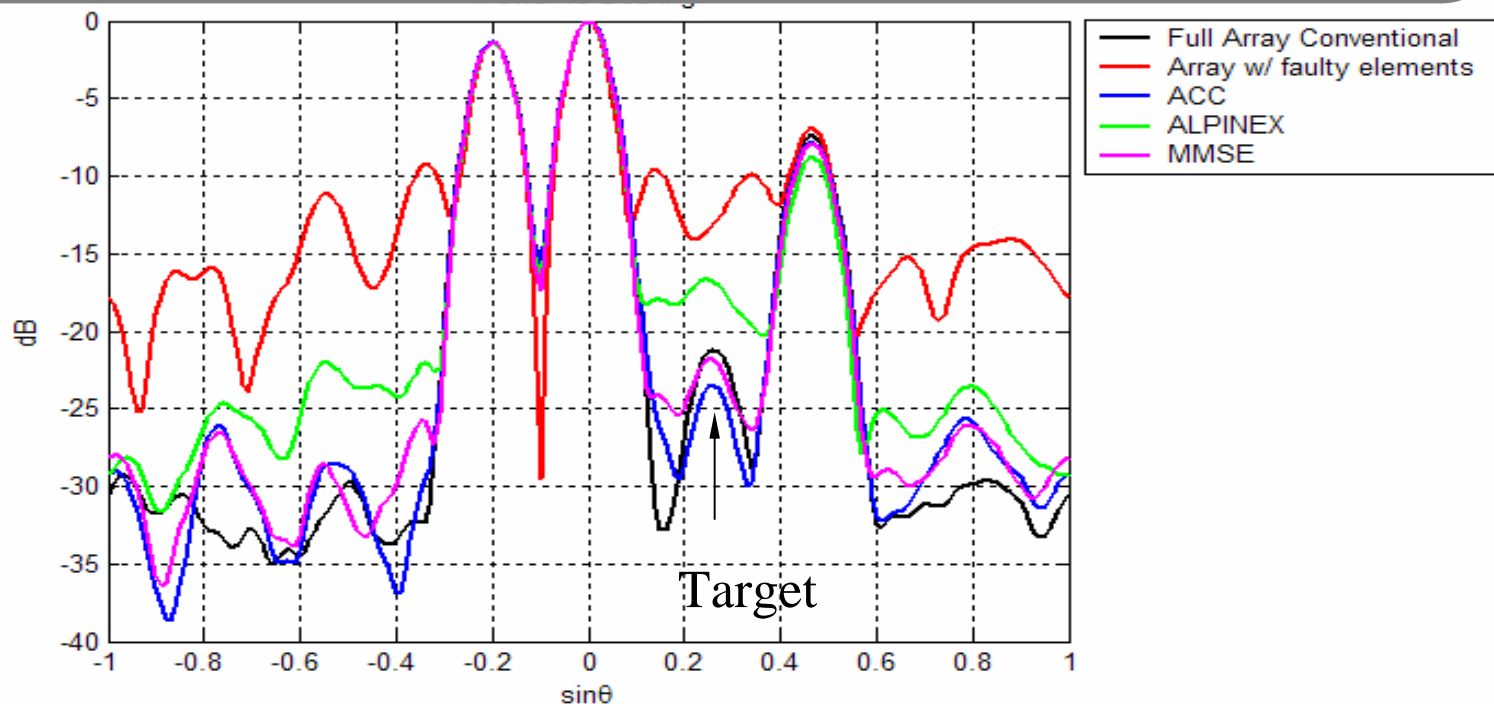
ACC vs. ABF with Limited Sample Support

- SINR vs. snapshot support simulation for 30 sensor array with 6 missing sensors in simulated multipath environment and target SNR = -10 dB.

- ACC comparable with dominant-mode rejection DMR-MVDR (Cox, Owsley) and superior to MMSE and SMI-MVDR for small sample support.

- Unlike all the ABF methods, ACC useful down just one snapshot.

SINR as a function of number of snapshots

Legend:
- Clairvoyant Full Array
- Array w/ faulty sensors
- ACC
- MMSE
- MVDR w/ DMR
- MVDR
- MVDR full array
- DMR full array

ACC Performance vs. Conventional Interpolation

- Single snapshot ROC for –4 dB target (30 sensor array with 6 missing) in simulated multipath environment (left). PD vs. SNR @ PFA=0.04 (right).

- ACC slightly outperforms MMSE at all SNR's.

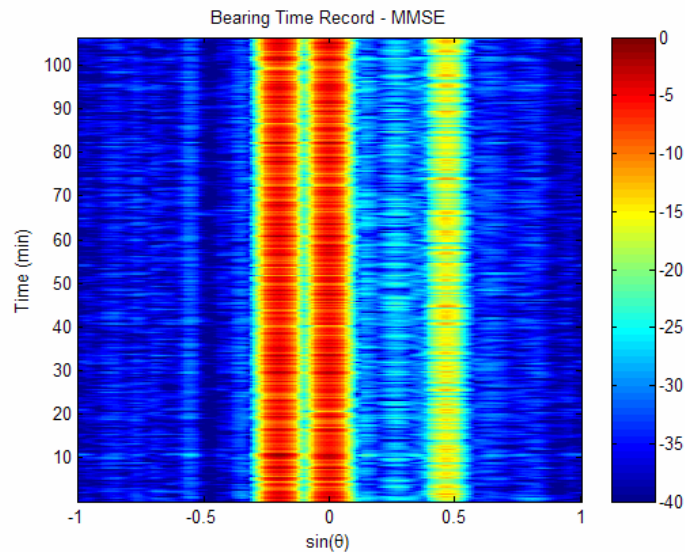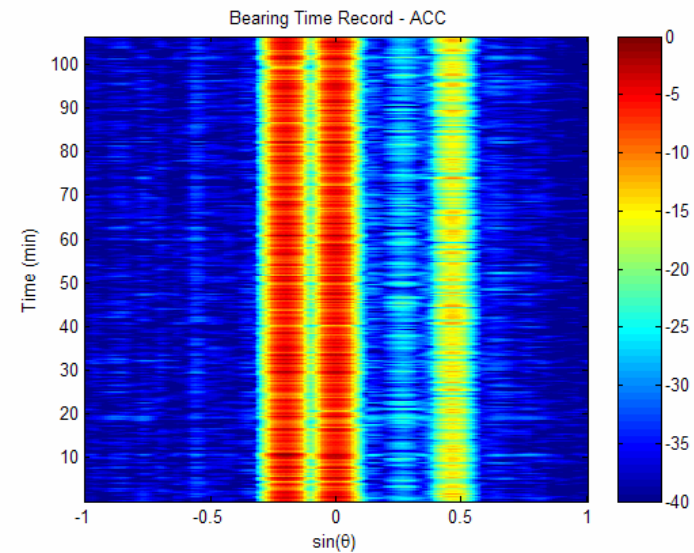- ALPINEX (Swinger and Walker) degrades significantly in correlated multipath.
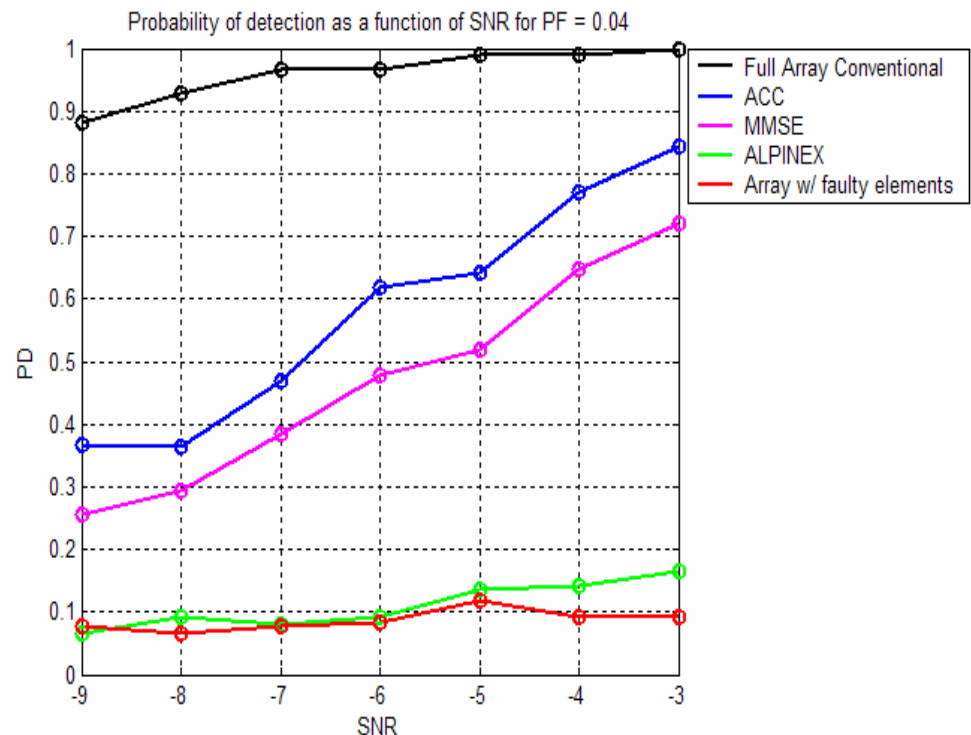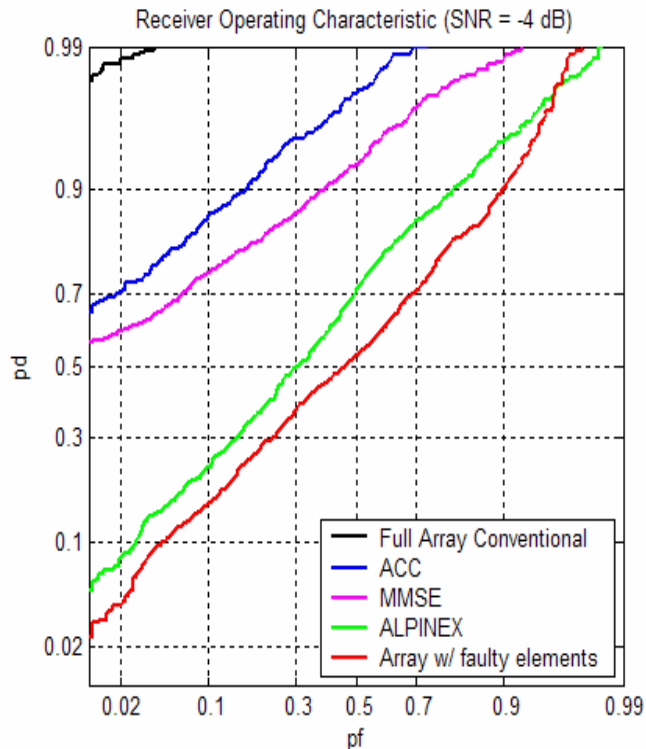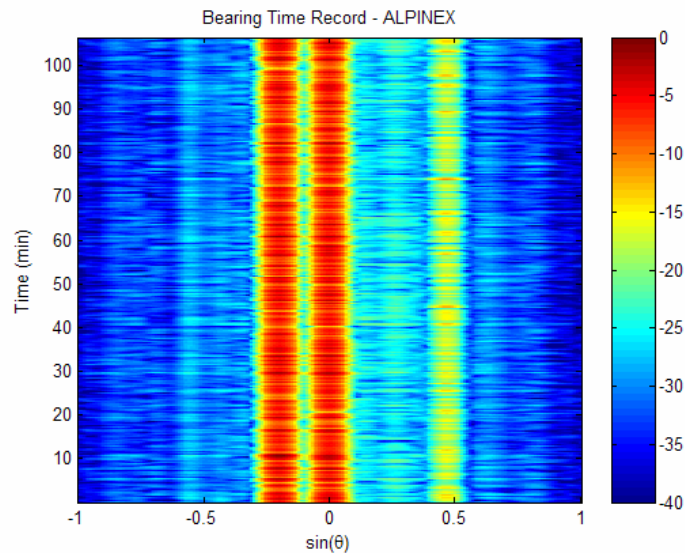
Simulated BTR for ACC vs. Clairvoyant Beamformer
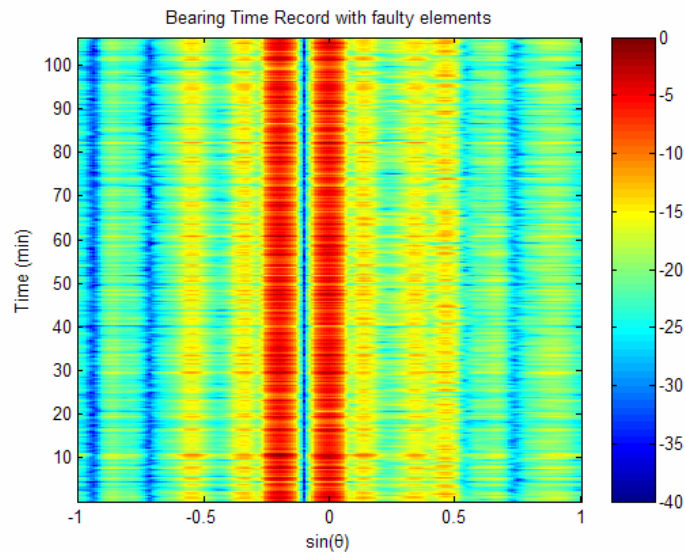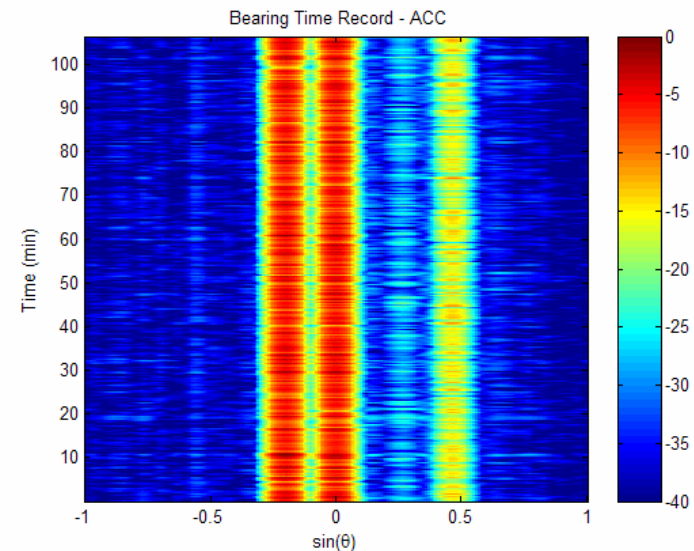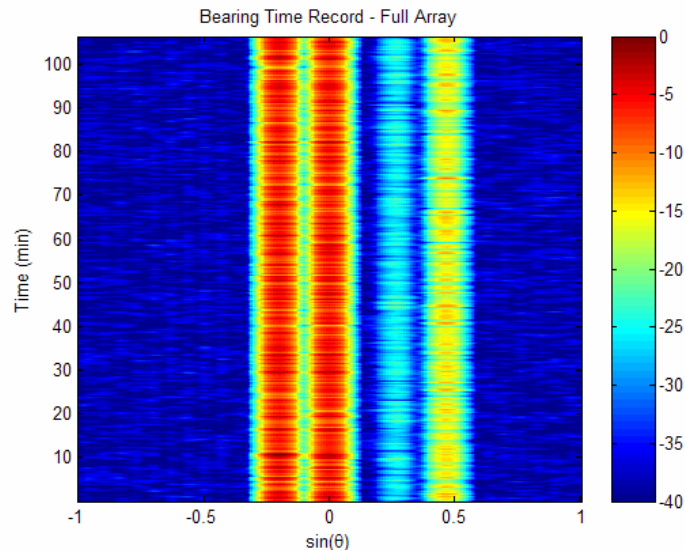
- Bearing-time records (BTR's) counterclockwise from above: Single-snapshot ACC, Clairvoyant CBF, and CBF with zeroed elements. Target at $\sin(\theta) = 0.27$ @ 0 dB SNR.

- ACC clearly unmasks target in the presence of resolvable interference.

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Simulated ACC vs. Interpolated Beamformers

Bearing Time Record - ALPINEX

Bearing Time Record - ACC

Bearing Time Record - MMSE

- Bearing-time records (BTR's) counterclockwise from above: Single-snapshot ACC, ALPINEX, and MMSE CBF with zeroed elements. Target at $\sin(\theta) = 0.27$ @ 0 dB SNR.

- ACC sidelobe suppression better than ALPINEX with fewer "ghost" tracks apparent in MMSE solution.

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Real Data BTR for ACC vs. Clairvoyant Beamformer
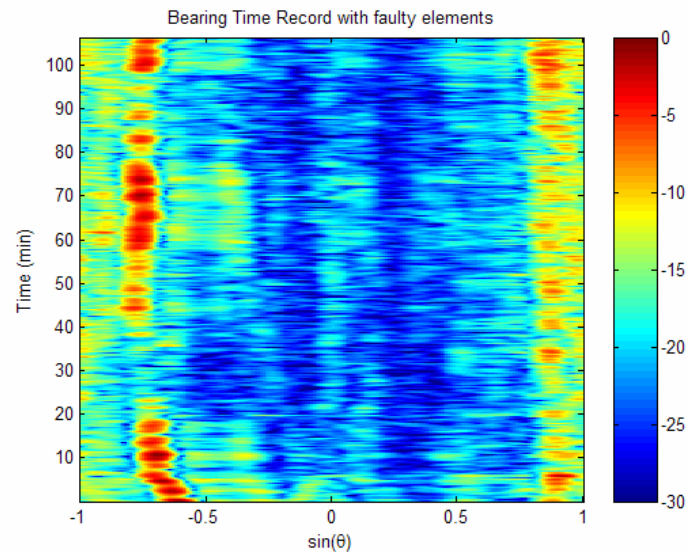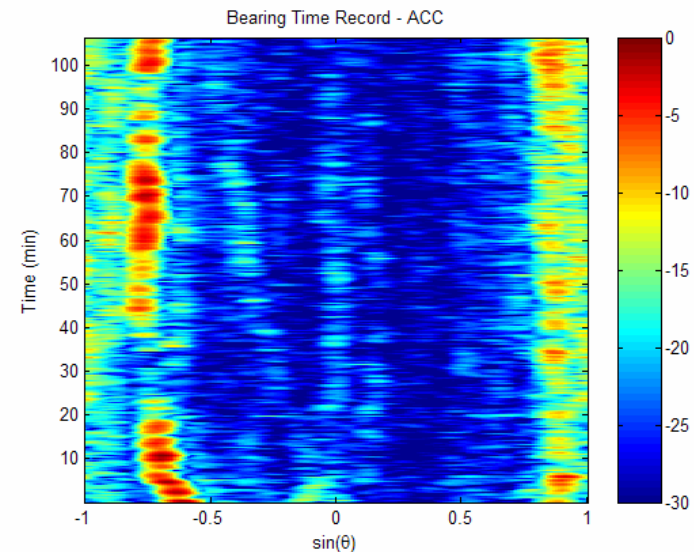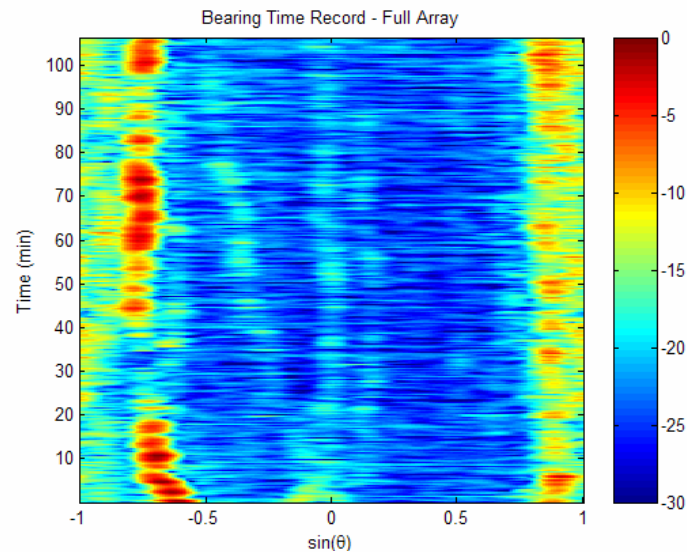
Bearing Time Record - Full Array

Bearing Time Record - ACC

Bearing Time Record with faulty elements

- Bearing-time records (BTR's) counterclockwise from above for real 30-element towed array data: Single-snapshot ACC, Clairvoyant CBF, and CBF with zeroed elements.

- Lack of ground-truth limits the ability to draw definitive conclusions.

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Real Data ACC vs. Interpolated Beamformers
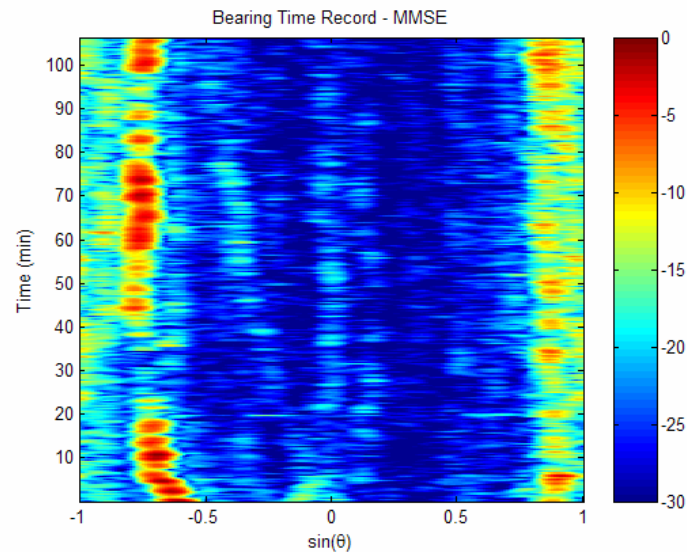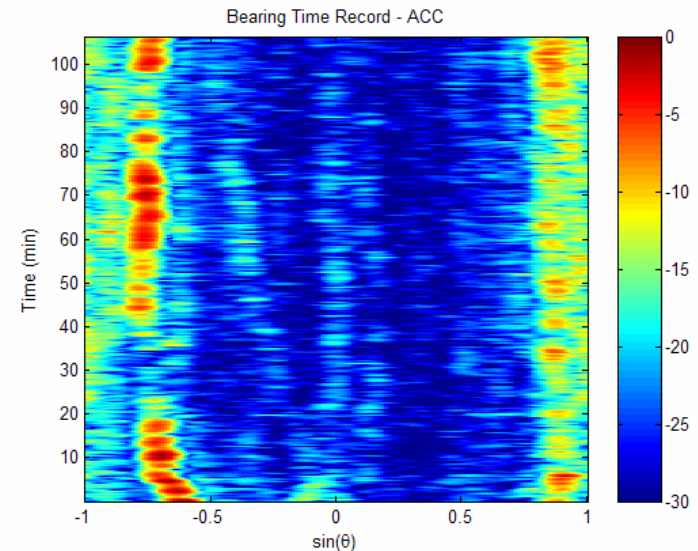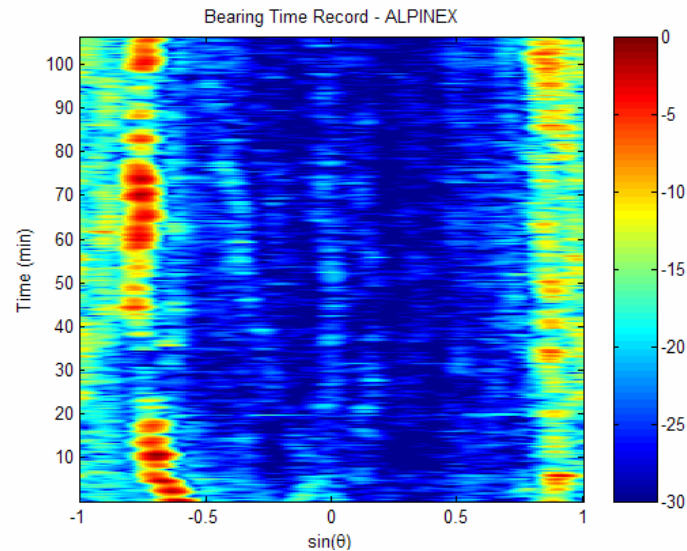
Bearing Time Record - ALPINEX

Bearing Time Record - ACC

Bearing Time Record - MMSE

- Bearing-time records (BTR's) counterclockwise from above for real 30-element towed array data: Single-snapshot ACC, ALPINEX, and MMSE CBF.

- ACC comparable to MMSE in this scenario.

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Summary and Further Work

- With increasing array apertures and channel count, mitigating the sidelobe impact of bad elements may become a critical problem.

- ABF snapshot support problem exacerbated with large arrays in dynamic littoral environments.

- MMSE interpolation methods not explicitly aimed at reducing leakage of strong interference into quiet look directions.

- ACC proposed as a means of compensating the entire array so that strong interferers made to look more like uncorrelated plane-waves which can then be suppressed by conventional beamformer shading.

- ACC outperforms MMSE and ALPINEX in terms of output SINR in low-snapshot support correlated multipath environments.

- Further work required to study performance in the presence of sensor calibration errors.

- ACC recently applied to 372 element OTH radar data with success in the presence of faulty elements.

- Possible applications of ACC to sonar data might include TB-29 or large bottom-mounted horizontal arrays.